Employee Bargaining Power, Inter-Firm Competition, and Equity-Based Compensation

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January 24th, 2014

Abstract

Principal-agent theory suggests that equity incentives should lead to greater effort from employees when effort is both costly and unobservable. However, due to free rider problems, this incentive effect may be limited when a firm employs a large number of workers. It is not clear then, why publicly-traded firms that employ many workers would choose to compensate their employees with company stock. I provide a possible explanation that is consistent with several empirical findings. Utilizing a model of employee bargaining power and inter-firm competition, I find a unique, pure strategy equilibrium where each competing firm offers an equity stake to its employees provided employee bargaining power is sufficiently low and inter-firm competition is sufficiently intense. This outcome arises because offering employees an equity stake improves wage efficiency and allows each firm to become more competitive with its rival. However, the equilibrium is a Prisoner’s Dilemma for the firms’ owners as they, and in some cases their employees, would be better off had the owners been able to commit to compensating employees with wages only.

I thank Viktoria Diser, Bjorn Jorgensen, Brian Mittendorf, N. V. Ramanan, and seminar participants at the 2014 AAA Management Accounting mid-year meeting, the 2013 Beyster Symposium, and the University of Toronto for helpful comments. I am grateful for funding from the University of Toronto and the Louis O. Kelso and Accurate Equity Fellowships which fund research on employee ownership. All errors are my own.
1. Introduction

Hölmstrom (1979) suggests that one way to elicit a high level of effort from an agent when agent effort is both unobservable and costly, is to make her compensation contingent on the firm’s profits. However, as Hölmstrom (1982), Huddart and Liang (2005), and others note, when too many employees are incentivized with contingent pay, a free-rider issue arises that may cause the incentive effect of contingent pay to be muted. Specifically, as the number of employees increases, individual workers may not exert extra effort because they can make only $1/N$ difference to the firm’s profits and $1/N$ becomes very small when $N$, the number of employees, becomes very large. In such a setting, as Oyer (2004) eloquently suggests, equity incentives may have no incentive effect, and the best response for all employees may be to not exert effort. Given this insight, it is perhaps puzzling that publicly-traded firms, which employ many workers, often compensate employees with equity in the company.

Several papers have put forth possible explanations to this puzzle. For example, Oyer (2004) suggests that a firm’s owners may compensate employees with equity stakes in order to index their compensation to outside options. In this respect, equity-based compensation might be useful as a tool for employee retention. Separately, Lazear (2004) and Arya and Mittendorf (2005) posit that a firm’s owners may compensate their employees with equity-based compensation for sorting purposes, where employees reveal information about either the firm or their own abilities by, “putting their money where their mouth is”.

I propose an additional factor that may lead a firm’s owners to compensate employees with equity in the company. I suggest that a firm’s owners may offer an equity stake to their

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1 These alternate explanations appear to be borne out empirically. Oyer and Shaffer (2005) consider three economic justifications for providing equity compensation to employees— incentive alignment, sorting, and employee retention—and find evidence consistent with the latter two. Core and Guay (2001) and Ittner, Lambert, and Larker (2003) also find evidence consistent with firms using equity-based pay for employee retention purposes.
employees in order to glean concessions on other facets of employee compensation. These concessions lower the firm’s cost base and make the firm not only more profitable, but also more competitive when facing a rival. I generate these inferences from a model of employee bargaining power and inter-firm competition. In the first stage of the model, a firm’s owners decide whether to compensate their employees with wages only or with wages and an equity stake in the company. In the second stage, rather than compensate employees only up to the point where employee participation is induced (i.e., the employees’ reservation wage), the owners and employees negotiate over the terms of the compensation (i.e., the level of wages and, in cases where equity compensation is offered, the percentage of the firm that will be given to employees). In this respect, the model allows for an inefficient labor market where employees extract above-market rents from the firm, contingent on their bargaining power.\(^2\) In the third and final stage, each firm makes production decisions as a function of negotiated employee compensation and the intensity of inter-firm competition.

I find that whenever a firm’s owners offer both wages and an equity stake to their employees, the firm is more profitable than if the owners offer employees wages only. This outcome arises because an equity stake in the company introduces a competing incentive for employees to maximize firm profits which, in turn, are decreasing in employee wages. While the firm’s profits are strictly higher when equity compensation is offered, the firm’s owners may or may not be better off. Specifically, while the firm is more profitable (i.e., the size of the pie is bigger), the owners’ remaining share of the profits is lower (i.e., the owner’s slice of the pie is smaller), as offering an equity stake to employees dilutes the owners’ position in the firm. In a setting with no competition, the costs to diluting the owners’ holdings in the company outweigh

\(^2\) Many factors such as unionization, local unemployment rates, and firm-borne employee switching costs can lead to inefficiencies in the labor market that allow employees to extract rents above the competitive market wage (see Bova, Dou, and Hope 2013, Lindbeck and Snower 1986, 2001).
the benefits to more efficient wages and greater firm profits. As a result, in a monopoly setting, owners have a strict disincentive to offer employees equity compensation.

In a competitive setting (i.e., the firm competes against a rival), the owners’ optimal strategy depends on the bargaining power of employees and the intensity of inter-firm competition. Notably, I find a unique, pure strategy equilibrium where each competing firm offers a wage and an equity stake to its employees (i.e., the employee ownership equilibrium) provided employee bargaining power is sufficiently low and inter-firm competition is sufficiently intense. This outcome arises because an equity stake generates more efficient wages which, in turn, reduces each firm’s cost base (as in the monopoly setting). This reduced cost base not only makes the firm more profitable, but also allows each firm to become more competitive with its rival. However, the equilibrium is a Prisoner’s Dilemma for each firm’s owners, as both firms’ owners would be better off had they been able to commit to offering their respective employees wages only. Moreover, in settings where competition is sufficiently intense, not only are the firm’s owners worse off by offering an equity stake, but so are the firm’s employees.

Importantly, the model’s results are consistent with several empirical findings. First, there is a large empirical literature that finds a positive correlation between equity-based compensation and firm productivity and profitability (see for example, Conte et al. 1996, Kim and Ouimet 2013, Jones and Kato 1995). A plausible explanation for this outcome is that equity compensation leads to greater worker effort which in turn drives greater firm production and profits. An alternate explanation however can be provided by the model. Specifically, when equity compensation is offered, employees also agree to more efficient wages. More efficient wages lead to a lower cost base for each firm, which in turn leads to greater production and
higher profits. Thus, even absent an incentive effect related to worker effort, compensating employees with equity in the company may lead to greater output and higher profits.

Second, there is mixed evidence regarding the firm and stakeholder benefits to adopting employee ownership plans in publicly-traded companies (e.g., Blasi, Conte, and Kruse 1996). In particular, there have been several puzzling findings related to the size of employee equity stakes and various stakeholder outcomes. For example, Kim and Ouimet (2013) suggest that when a firm compensates employees with small equity stakes in the company, not only is the firm more profitable, but employees extract more of the surplus generated following the equity grant. Conversely, Kim and Ouimet (2013) and Faleye et al. (2006) find that, when employees are granted a large equity stake in the company, the effects of employee ownership are muted and, often, all parties are worse off. Based on these empirical observations, a first question might be to ask why firm and stakeholder outcomes vary with the size of employee equity stakes. A second question might be to ask how the decision to grant large equity stakes arises endogenously, if large equity stakes appear to make both owners and employees worse off.

A possible explanation arises by assessing two important comparative statics in the model. Specifically, any time the employee ownership equilibrium is supported, the optimal equity stake (wage) for each firm is increasing (decreasing) in the competitiveness of the market. So, when a firm has a near monopoly in its industry, we can expect employees to be compensated with a small equity stake and a higher wage. Conversely, when the firm faces intense competition from a rival, we can expect employees to be compensated with a large equity stake and lower wages.

When equity stakes are fairly small (i.e., settings where competition is more muted), I find that employees are better off with equity compensation than with wages only. Employees
are better off because their wages are still relatively large and they enjoy a small share of the firm’s profits when the firm’s profits are also relatively large due to less competition. This result is consistent with the finding that when employees own smaller equity stakes in the company, they often enjoy more of the ensuing surplus (i.e., Kim and Ouimet 2013).

Conversely, when equity stakes are large (i.e., settings where competition is more intense), I find that, not only are the firm’s owners worse off by offering an equity stake, but so too are the firm’s employees. This outcome ensues as wages are relatively low and, although employees enjoy a larger share of the firm’s profits via a larger equity stake, the firm’s profits are also comparatively low because competitive pressures are high. Interestingly, the model nevertheless suggests that we should expect large employee equity stakes to arise endogenously in highly competitive settings, despite owners and employees being worse off than if owners offered wages only.

Finally, I provide the 2009 contract negotiations between the United Auto Workers (UAW) and General Motors, Ford, and Chrysler, as a practical example of the tensions described in the model. It is important to note that the auto industry is characterized by intense inter-industry competition and that the UAW entered this particular contract negotiation with reduced bargaining power due to the Financial Crisis. Given these points, several outcomes that followed the negotiations appear to be consistent with the results generated by the model. First, employees made steep concessions to their cash wages (e.g., entry position hourly wages were reduced to $14/ hour) in return for large equity stakes in the company and profit sharing arrangements. Second, all three firms, as opposed to only one firm, agreed to contracts with

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3 The reduced negotiation leverage was driven by the U.S. government’s threat to push G.M. and Chrysler in to bankruptcy if the UAW did not make concessions.

lower wages and benefits but larger equity stakes and profit sharing.\(^5\) Third, UAW members became some of the largest employee owners of publicly-traded stock in the U.S.\(^6\) Fourth, the wage concessions appear to have increased both production and profits at all three firms.

The combined analysis highlights employee bargaining power and inter-firm competition as two important factors that may drive both the existence and the level of employee ownership in a firm and, more broadly, an industry. The model’s results also provide several insights that may explain that documented variation in stakeholder outcomes arising from owners compensating their employees with equity, including a potential explanation for how employee ownership arises endogenously even when both owners and employees would be better off had employees been compensated with wages only. The paper proceeds as follows: Section 2 presents the model and results, Section 3 provides limitations and implications, and Section 4 concludes.

2. Model

The analysis encompasses a single period model set in three stages. To highlight the importance of competition in the equilibria, I present two settings – a benchmark setting with no competition (i.e., one firm has a monopoly) and a focal setting which incorporates competition (i.e., two firms compete in a duopoly). For brevity, I describe the stages of the game for the duopoly setting, but note that they are identical to the monopoly setting, save that in the monopoly setting there is one firm instead of two.

\(^5\) The model predicts that, in equilibrium, either each firm offers wages and an equity stake or each firm offer wages only to its employees.

\(^6\) Employees own roughly 17.5% of G.M.’s shares, over 10% of Ford’s shares, and 55% of Chrysler’s shares http://www.heritage.org/research/reports/2012/06/auto-bailout-or-uaw-bailout-taxpayer-losses-came-from-subsidizing-union-compensation.
In the first stage of the game, each firm’s owners decide whether to offer their respective employees wages only or wages and an equity stake in the company. In the second stage of the game, each firm’s owners and employees negotiate over the terms of the compensation (i.e., the level of wages and, in cases where equity compensation is offered, the percentage of the firm that will be given to employees) with the negotiated outcome driven by the degree to which employees have bargaining power over their respective owners. In this respect, the model allows for an inefficient labor market where employees can extract above-market rents from the firm, contingent on their bargaining power. In the final stage of the game, each firm sets production to maximize profits, prices are realized, and profits accrue to owners and employees provided employees have an equity stake in the firm. The game’s three stages are represented in Figure 1.

![Figure 1 Timeline](image)

Each firm’s owners simultaneously chooses whether to compensate their employees with wages only or wages and an equity stake. Each firm’s owners and employees simultaneously negotiate over the level of wages, and in cases where an equity stake is offered, the level of equity given to employees. Quantities are simultaneously chosen by each firm, consumers purchase, and each firm realizes its profits – a portion, , of which accrue to employees.

2.1 Monopoly Setting

To provide a benchmark setting, I first assess a model without competition. In this setting, the timeline is identical to the one described in Figure 1, but the focal firm faces no rival competitor. Consumer demand for the firm’s product is represented by the following linear (inverse) demand function, , where is the firm’s product price and is the quantity
produced by the firm. I assume that the firm has one input to production, labor, and that there are constant returns to scale so that one unit of labor is used to produce one unit of product. The firm pays a wage, \( w \), where \( w \geq 0 \), for one unit of labor and the competitive market wage per unit is, \( c \). Finally, \( c < 1 \), where 1 represents the demand intercept.

To generate the subgame perfect equilibrium, I work backward in the game. In the third stage of the game, given the negotiated employee wage, \( w \), the firm’s chooses \( q \) to maximize its profit function:

\[
Max \Pi = q[1-q] - q[w]
\]  

(1)

The first order condition in (1) with respect to \( q \) yields \( q(w) \), the optimal quantity as a function of employee wages:

\[
q(w) = \frac{1-w}{2}
\]  

(2)

Plugging (2) into (1) yields firm profits as a function of the employee wage, \( \Pi(w) \). In the second stage, the firm’s owners and its employees negotiate over the level of wage, \( w \), and in cases where the owners offer an equity stake in the firm, the level of equity stake, \( z \), given \( \Pi(w) \). The firm’s owners seek to maximize, \( S(w, z) \), the residual profits that do not accrue to employees:

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7 This type of demand function arises when consumers maximize a quadratic utility function subject to a budget constraint as in Singh and Vives (1984)
8 This modelling assumption maps in to the UAW example described in the Introduction. Specifically, UAW employees are paid an hourly wage, \( w \), of $14/hour for direct labor, and direct labor hours are significant inputs in to production at GM, Ford, and Chrysler.
9 When an equity stake is offered to employees, employees are compensated with cash wages based on the piece rate wage, \( w \), that is set before production, and their share of company profits, \( z \), that is garnered after quantity is set and profits are realized. Thus employee equity stakes, if they are offered, do not impact the cost of production.
The firm’s employees seek to maximize, \( E(w, z) \), the sum of their wage and equity-based compensation:

\[
E(w, z) = q(w)[w - c] + z[\Pi(w)]
\]  

The employee participation constraint is \( E(w, z) \geq 0 \).\(^{10}\) To generate the optimal compensation levels in the second stage, the owners and employees set the wage, \( w \), and in cases where the owners offer an equity stake in the firm, the level of equity stake, \( z \), to maximize the generalized Nash product below:

\[
[E(w, z)]^\beta [S(w, z)]^{1-\beta}
\]

\( \beta \) represents the employees’ negotiation leverage and \( \beta \in (0,1) \).\(^{11}\) As \( \beta \to 1 \), employees glean next to all of the negotiation leverage and are effectively the monopolist supplier of labor. As

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\(^{10}\) While I assume \( w \geq 0 \), I do not assume \( w \geq c \). Thus, I allow for \( w < c \). \( w < c \) may arise provided employees are compensated with a large enough equity stake in the company, such that \( E(w, z) \geq 0 \). Such a contract would be an example of financial bootstrapping (i.e., a contract where employees forgo cash wages in return for an equity stake in the company).

\(^{11}\) In (5), employees negotiate to maximize the sum of aggregate wages and equity stakes across all employees. This modelling choice is consistent with evidence that bargaining often takes place between owners and employee representatives that negotiate on behalf of a group of employees. Additionally, as the model assumes an inefficient labor market, I note that the market for labor is often inefficient in precisely those settings where representatives negotiate on behalf of a group of employees (e.g., collective bargaining groups).
\( \beta \to 0 \), employees glean next to none of the negotiation leverage and the market for labor approaches perfect competition.

The outcome in the game’s second stage depends on whether the firm’s owners offer wages only or wages and an equity stake to their employees in the first stage. As a result, I derive the optimal outcomes negotiated in the second stage conditional on the type of compensation offered in the first stage.

2.1.1 The firm’s owners offer wages only

If the firm’s owners offer to compensate employees with wages only, I set \( z = 0 \) in (5) and both parties choose \( w \) to maximize the generalized Nash product in (5).\(^{12}\) The first order condition of (5) with respect to \( w \) yields \( w(\beta) \). Substituting \( w(\beta) \) and the firm’s optimal quantity from (2) into (1), (3), and (4) yields the firm’s equilibrium profits, \( \Pi^w(\beta) \), the owners’ share of the profits, \( S^w(\beta) \), and employee compensation, \( E^w(\beta) \), respectively. The equilibrium outcomes when owners offer to compensate their employees with wages only are summarized below:

\[
\begin{align*}
  w^w(\beta) &= \frac{2c + (1-c)\beta}{2}, \\
  q^w(\beta) &= \frac{(2-\beta)(1-c)}{4}, \\
  E^w(\beta) &= \frac{\beta(2-\beta)(1-c)^2}{8} \\
  S^w(\beta) &= \Pi^w(\beta) = \frac{(2-\beta)^2(1-c)^2}{16}
\end{align*}
\]

As expected, in a model without equity ownership, wages (firm profits) are increasing (decreasing) in the employees’ negotiation leverage.

\(^{12}\) I denote any setting where a firm’s owners compensate their employees with wages only with the superscript, \( w \).
2.1.2 The firm’s owners offer wages and an equity stake in the firm

If the firm’s owners offer to compensate employees with wages and an equity stake in the firm in the first stage, the contract that arises as a result of the bargaining process is given by the parameter values $w$ and $z$ that maximize the generalized Nash product in (5).\(^{13}\) Solving the first order conditions of (5) with respect to $w$ and $z$ yields the employees’ wage, $w$, and the employees’ equity stake, $z(\beta)$. Substituting the wage, $w$, and optimal quantity from (2), into (1), (3), and (4) and the equity stake, $z(\beta)$, into (3) and (4) yields the firm’s equilibrium profits, $\Pi^z$, owners’ share of the profits, $S^z(\beta)$, and employee compensation, $E^z(\beta)$, respectively. The equilibrium outcomes when the firm’s owners offer to compensate employees with both a wage and an equity stake are summarized below:

$$
\begin{align*}
  z^z(\beta) &= \beta, \quad w^z = c, \quad q^z = \frac{1-c}{2}, \quad S^z(\beta) = \frac{(1-\beta)(1-c)^2}{4}, \\
  E^z &= \beta(1-c)^2, \quad \Pi^z = \frac{(1-c)^2}{4}
\end{align*}
$$

Note that the wage is strictly lower (i.e., employees are paid the competitive wage irrespective of bargaining power) when a firm’s owners offer to give employees an equity stake in the company. This outcome arises as an equity stake incents employees to focus on maximizing firm profits, and firm profits are decreasing in employee wages. Note also, that the

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\(^{13}\) I denote any setting where a firm’s owners compensate their employees with wages and an equity stake with the superscript, $z$. 
optimal compensation structure when a firm and its employees negotiate over both an equity stake and wages is a two-part tariff, where the employees are paid the reservation wage and the ensuing profits are divided proportionally depending on which party has more bargaining power.\textsuperscript{14}

\textbf{PROPOSITION 1.} In a market with no competition:

\begin{enumerate}[
\text{i.})
\item owners never offer their employees an equity stake in the firm.
\item the firm is more profitable and employees are strictly better off when owners compensate employees with both wages and an equity stake than when owners compensate employees with wages only.
\end{enumerate}

As wages are strictly lower in a model where employees have an equity stake, it is intuitive to see that the firm is more profitable due to a lower cost base. The same however is not true for the share of profits that accrue to the firm’s owners. On the one hand, providing employees with an equity stake in the company reduces employee wages to the reservation wage and leads to greater firm profits than if the firm’s owners compensated employees with wages only. On the other hand, compensating employees with an equity stake dilutes the owners’ equity position, meaning that they accrue a smaller portion of firm profits. When the firm has a monopoly, this latter force dominates for all $\beta \in (0,1)$, and owners are worse off if they offer an equity stake. Thus, despite the firm being more profitable and the employees being better off when compensated with an equity stake, the owners will never opt to offer equity compensation

\textsuperscript{14} For another example of generalized Nash bargaining over a two-part tariff, see Arya and Mittendorf (2013).
to their employees in a setting with no competition.\textsuperscript{15} A natural conclusion from this result is that we should \textit{not} expect a firm to compensate its employees with equity in order to make their wage bill more efficient. In the next section, I reassess this conclusion in a competitive setting.

2.2 \textit{Competitive Setting}

In the competitive setting, two firms compete in a duopoly. Consumer demand for each firm’s product is represented by a linear (inverse) demand function $P_i = 1 - q_i - kq_j$ where $P_i$ is firm $i$’s product price, $q_i$ is the quantity produced by firm $i$, and $i, j = 1, 2, i \neq j$. $k$ is the substitutability parameter and $k \in (0,1]$. When $k = 1$, the firms produce perfect substitutes. As $k \to 0$, each firm has a near monopoly for its product. Thus, competition intensity is increasing in $k$. As in the monopoly setting, I assume that each firm has one input, labor, and that there are constant returns to scale so that one unit of labor is used to produce one unit of product.\textsuperscript{16} The cost to each firm for one unit of labor is the wage, $w_i$, where $w_i \geq 0$ and the competitive market wage per unit is $c$. I also assume that $c \geq c^*(k) = \frac{k^2}{4 + 2k}$, as this assumption guarantees $w_i \geq 0$ irrespective of the type of compensation offered by each firm’s owners in the game’s first stage. Finally, $c < 1$, where 1 represents the size of the market.

\textsuperscript{15} Although an equity stake should not be granted to employees in order to glean wage concessions in a monopoly setting, this does not preclude the possibility that owners of a monopoly might provide equity-based compensation for other reasons.

\textsuperscript{16} As employees are input suppliers, the analysis also contributes to the literature that assesses the effects of competition on supply chain contracts (see for example, Arya and Mittendorf 2007 and Arya, Mittendorf, and Yoon 2013).
To generate the subgame perfect equilibrium, I work backward in the game. In the third stage of the game, firm $i$ chooses $q_i$ to maximize its profits in (8), given its negotiated employee wage, $w_i$, and $q_j$:

$$\max_{q_i} \Pi_i = q_i[1 - q_i - k q_j] - q_i[w_j], \quad i, j = 1, 2, i \neq j$$

(8)

Solving the two first order conditions in (8) yields the symmetric equilibrium quantities as a function of each firm’s employee wages and product substitutability:

$$q_i(w_i, w_j, k) = \frac{2 - k + 2w_i - kw_j}{4 - k^2}, \quad i, j = 1, 2, i \neq j$$

(9)

Substituting the quantities from (9) into (8), yields firm profits as a function of each firm’s employee wages and product substitutability, $\Pi_i(w_i, w_j, k)$.

In the second stage, as in the monopoly setting, each firm’s owners and their respective employees negotiate over the level of wage, $w_i$, and in cases where the owners offer an equity stake in the firm, the level of equity stake, $z_i$, given expected $\Pi_i(w_i, w_j, k)$. Each firm’s owners seek to maximize, $S_i(z_i, w_i, w_j, k)$, the residual profits that do not accrue to their employees:

$$S_i(z_i, w_i, w_j, k) = (1 - z_i)\Pi_i(w_i, w_j, k), \quad i, j = 1, 2, i \neq j$$

(10)
Each firm’s employees seek to maximize the sum of their wages and equity stakes, $E_i(z_i, w_i, w_j, k)$:

$$E_i(z_i, w_i, w_j, k) = q_i(w_i, w_j, k)[w_i - c] + z_i\Pi_i(w_i, w_j, k) \quad i, j = 1, 2, i \neq j$$

(11)

To generate optimal compensation levels in the second stage, each firm’s owners and employees set the wage, $w_i$, and in cases where the owners offer an equity stake in the firm, equity stake, $z_i$, to maximize the generalized Nash product below:

$$[E(z_i, w_i, w_j, k)]^\beta[S_i(z_i, w_i, w_j, k)]^{1-\beta} \quad i, j = 1, 2, i \neq j$$

(12)

$\beta$ represents the employees’ negotiation leverage and $\beta \in (0, 1)$. As in the monopoly setting, each firm’s optimal compensation structure generated in the second stage is driven by the type of compensation offered by the owners in the first stage. However, unlike the monopoly setting, each firm’s optimal compensation structure also hinges on the type of compensation offered by the rival firm’s owners. Thus, there are three possible strategy sets in the first stage: each firm’s owners offer wages only; each firm’s owners offer both wages and an equity stake; and one firm’s owners offer wages only and one firm’s owners offer both wages and an equity stake. I assess the optimal outcomes negotiated in the second stage conditional on each of the possible strategy sets in the first stage.

2.2.1 Both firms’ owners offer wages only
If both sets of owners offer wages only in the first stage, then neither firm’s owners offer an equity stake to their employees and $z_1 = z_2 = 0$ in (12). In this setting, each owner/employee pairing simultaneously solves the first order condition of (12) with respect to $w_i$, yielding each employee group’s optimal wage, $w_i(\beta, k)$. Substituting each firm’s optimal quantity from (9) and wage, $w_i(\beta, k)$, into (8), (10), and (11) yields each firm’s profits, $\Pi_i^{ww}(\beta, k)$, owners’ share of profits, $S_i^{ww}(\beta)$, and employee compensation $E_i^{ww}(\beta)$, respectively. The equilibrium outcomes when both sets of owners offer wages only to their employees are summarized below:

$$w_1^{ww}(\beta, k) = w_2^{ww}(\beta, k) = \frac{4c + \beta(2-2c+k)}{4-\beta k}, q_1^{ww}(\beta, k) = q_2^{ww}(\beta, k) = \frac{2(2-\beta)(1-c)}{(2+k)(4-\beta k)}$$

$$E_1^{ww}(\beta, k) = E_2^{ww}(\beta, k) = \frac{2(2-\beta)(2-k)\beta(1-c)^2}{(2+k)^2(4-\beta k)^2},$$

$$S_1^{ww}(\beta, k) = S_2^{ww}(\beta, k) = \frac{4(2-\beta)^2(1-c)^2}{(2+k)^2(4-\beta k)^2}.$$

As in the one firm case, it is easy to see that employee wages (firm profits) are increasing (decreasing) in employee bargaining power when owners offer wages only to their employees.

2.2.2 Both firms’ owners offer wages and an equity stake

If both sets of owners offer wages and an equity stake in the firm to their employees, each owner/employee pairing simultaneously solves the first order conditions of (12) with respect to $w_i$, and equity share, $z_i$, yielding the optimal wage, $w_i(\beta, k)$, and optimal equity stake, $z_i(\beta, k)$.
for each firm. Substituting each firm’s optimal quantity from (9) and wage $w_i(\beta, k)$ into (8), (10), and (11), and each firm’s equity stake $z_i(\beta, k)$ into (10) and (11) yields each firm’s optimal profits, $\Pi_i^z(\beta, k)$, owner’s share of the profits, $S_i^z(\beta)$, and employee compensation, $E_i^z(\beta)$, respectively. The equilibrium outcomes when both firms’ owners offer wages and equity stakes are summarized below:

\[
\begin{align*}
z_i^z(\beta, k) &= z_2^z(\beta, k) = \beta + \frac{(1 - \beta)k^2}{2}, \\
w_i^z(\beta, k) &= w_2^z(\beta, k) = \frac{2c(2 + k) - k^2}{4 + 2k - k^2}, \\
q_1^z(\beta, k) &= q_2^z(\beta, k) = \frac{2(1 - c)}{4 + 2k - k^2}, \\
E_i^z(\beta, k) &= E_2^z(\beta, k) = \frac{2\beta(1 - c)^2(2 - k^2)}{(4 + 2k - k^2)^2}, \\
S_1^z(\beta, k) &= S_2^z(\beta, k) = \frac{2(1 - \beta)(1 - c)^2(2 - k^2)}{(4 + 2k - k^2)^2}, \\
\Pi_1^z(\beta, k) &= \Pi_2^z(\beta, k) = \frac{4(1 - c)^2}{(4 + 2k - k^2)^2}.
\end{align*}
\]  

(14)

**Lemma 1.** When both firms’ owners offer wages and equity stakes to employees, equity stakes and wages are increasing and decreasing in product substitutability, respectively.

The outcomes in Lemma 1 are driven by two forces. First, the more competitive the setting, the more important it is for the focal firm to glean a competitive advantage over its competitor. Second, the larger the equity stake, the more employees focus on maximizing firm profits which are decreasing in wages. Thus, it is perhaps unsurprising that in highly competitive settings, we observe employees with the highest equity stakes and the lowest wages. Finally,
note that an implicit takeaway from the analysis is that whenever both firms’ owners offer equity stakes to their employees, equity stakes substitute, as opposed to complement, wages. This substitution effect is at its largest when competition is at its most intense and in turn, equity stakes (wages) are at their highest (lowest). This outcome maps in to the Kim and Ouimet (2013) finding that wages decreases when employees are compensated with large equity stakes – a result driven, in part, by the substitution of cash wages with company stock.

2.2.3 One firm’s owners offer wages only, one firm’s owners offer both wages and an equity stake.

Finally, I assess a setting where one firm’s owners compensate employees with wages only and the other firm’s owners compensate employees with wages and an equity stake. Without loss of generality, I assume that the owners of firm 1 offer their employees both wages and an equity stake and the owners of firm 2 offer their employees wages only. Thus, I set \( z_2 = 0 \) for firm 2 in (12). Simultaneously, firm 1’s owner/employee pairing solves the first order conditions of (12) with respect to wage, \( w_1 \), and equity share, \( z_1 \), and firm 2’s owner/employees pairing solves the first order condition of (12) with respect to wage, \( w_2 \). This yields optimal wages, \( w_1(\beta, k) \) and \( w_2(\beta, k) \) and optimal equity stake, \( z_1(\beta, k) \). Substituting each firm’s optimal quantities from (9) and each firm’s optimal wages \( w_1(\beta, k) \) and \( w_2(\beta, k) \) into (8), (10), and (11) along with firm 1’s optimal equity stake \( z_1(\beta, k) \) into firm 1’s (10) and (11), yields each firm’s profits, \( \Pi_i^{zw}(\beta, k) \), each owners’ share of the profits, \( S_i^{zw}(\beta) \), and each employee group’s compensation, \( E_i^{zw}(\beta) \), respectively. The equilibrium outcomes when firm 1’s owners
offer wages and an equity stake to its employees and firm 2’s owners offer wages only to its employees are summarized below:

\[
\begin{align*}
    z_w^1(\beta, k) &= \beta + \frac{(1 - \beta)k^2}{2}, \\
    w_2^w(\beta, k) &= \frac{32c - 8(1 + c)k^2 + 2(2 - \beta)(1 - c)k^3 + \beta k^4}{32 - 16k^2 + \beta k^4},
\end{align*}
\]

\[
\begin{align*}
    w_2^w(\beta, k) &= \frac{16c(2^2 - k^2) + \beta((2 - k)(2 + k)(4 - 2k - k^2) - 2c(8 - 4k - 4k^2 + k^3))}{32 - 16k^2 + \beta k^4},
\end{align*}
\]

\[
\begin{align*}
    q_i^w(\beta, k) &= \frac{2(1 - c)(2 - k)(4 + \beta k)}{32 - 16k^2 + \beta k^4}, q_i^w(\beta, k) = \frac{2(1 - c)(2 - \beta)(4 - 2k - k^2)}{32 - 16k^2 + \beta k^4},
\end{align*}
\]

\[
\begin{align*}
    E_1^w(\beta, k) &= \frac{2\beta(1 - c)^2(2 - k)^2(4 + \beta k)^2(2 - k^2)}{(32 - 16k^2 - \beta k^4)^2},
\end{align*}
\]

\[
\begin{align*}
    E_2^w(\beta, k) &= \frac{2\beta(2 - \beta)(1 - c)^2(4 - 2k - k^2)^2(4 - k^2)}{(32 - 16k^2 - \beta k^4)^2},
\end{align*}
\]

\[
\begin{align*}
    S_1^w(\beta, k) &= \frac{2(1 - \beta)(1 - c)^2(2 - k)^2(4 + \beta k)^2(2 - k^2)}{(32 - 16k^2 - \beta k^4)^2},
\end{align*}
\]

\[
\begin{align*}
    \Pi_1^w(\beta, k) &= \frac{4(1 - c)^2(2 - k)^2(4 + \beta k)^2}{(32 - 16k^2 - \beta k^4)^2},
\end{align*}
\]

\[
\begin{align*}
    S_2^w(\beta, k) &= \Pi_2^w(\beta, k) = \frac{4(1 - c)^2(4 - 2k - k^2)^2(2 - \beta)^2}{(32 - 16k^2 - \beta k^4)^2}.
\end{align*}
\]

Having defined the optimal outcomes contingent on the type of compensation chosen by the owners in the first stage, I next define the parameter space over which both firms’ owners offer a wage and an equity stake to their respective employees.
PROPOSITION 2.

i.) If $\beta < \beta^*(k)$, there is a unique, pure strategy equilibrium where both sets of owners offer wages and equity stakes to their employees.

ii.) The equilibrium in i.) is a Prisoner’s Dilemma for each firm’s owners, as both sets of owners would be better off had they both been able to commit to offering wages only.

iii.) Firm profits are strictly higher when both firms’ owners offer wages and equity stakes to their employees than when both firms’ owners offer wages only.

If $\beta < \beta^*(k)$ and both firms’ owners offer employees wages only, the best response for the focal firm’s owners is to deviate from offering wages only to offering wages and an equity stake. By doing so, the focal firm gleans concessions in the piece rate wage, $w$, from its employees and reduces its cost base. This reduced cost base gives the focal firm a competitive cost advantage over its rival which makes the focal firm more profitable and, importantly, its owners better off despite diluting their holdings. However, when $\beta < \beta^*(k)$, the competing firm’s owners’ best response to the focal firm’s owners offering their employees an additional equity stake is to also offer their employees an additional equity stake. Doing so leads to parity in wages across both firms, resulting in neither firm having a competitive cost advantage over the other. Taken together, when $\beta < \beta^*(k)$, there is a unique, pure strategy equilibrium where each firm’s owners offer their employees wages and an equity stake in the company.\(^{17}\) The shaded

\(^{17}\) When $\beta > \beta^*(k)$, there is portion of the parameter space where two pure strategy equilibria are supported: One pure strategy equilibrium where both firm’s owners offer wages only, and one pure strategy equilibrium where both firm’s owners offer wages and an equity stake. However, as Proposition 2, claim 3 illustrates, the wages only equilibrium Pareto dominates the wages and equity stake equilibrium. As a result, when $\beta > \beta^*(k)$, I assume that both owners will choose the wages only equilibrium as it is either a unique, pure strategy equilibrium or a Pareto dominant, pure strategy equilibrium.
area in Figure 2 illustrates the parameter space over which both firm’s owners offer wages and an equity stake to their employees and employee wages are non-negative. I denote this parameter space, *the employee ownership equilibrium*.

Both firms are strictly more profitable when the employee ownership equilibrium is supported because equity stakes drive lower input costs. The lower costs, in turn, reduce double marginalization across the channel, and lead to greater profits. However, both firms’ owners are strictly worse off by offering equity stakes than had they both offered wages only. As in the monopoly benchmark case, although profits are greater when owners compensate employees with equity in the company, the owners’ share of those profits are smaller, and it is this latter force that dominates. This leads to the interesting result that both sets of owners offer their respective employees an equity stake in the firm, despite the owners being better off had they been able to commit to offering employees wages only.

**Figure 2**
*Parameter space over which wages are strictly positive and both firms’ owners offer equity compensation*
Proposition 1 provides insight into not only the types of firms where employee ownership should be more prominent, but also the types of firms where equity compensation should not be offered. For example, in firms where employees have greater bargaining power (i.e., $\beta > \beta'(k)$), a focal firm’s owners best response to a competing firm’s owners offering wages only is to also offer wages only. To understand why this is a best response, we can assess an extreme setting where employees have nearly all of the negotiation power (i.e., $\beta \to 1$). In this setting, offering an equity stake leads to a much lower cost base, but also means that nearly all of the firm’s profits will accrue to employees because employees will be compensated with nearly all of the firm’s equity. In such a setting, both firms’ owners would be better off offering wages only, having a higher cost base, and retaining 100% of (albeit smaller) profits. Interestingly, this insight provides a potential explanation for why we do not commonly observe equity-based compensation in highly unionized settings. Unionized firms tend to have more negotiation leverage over the firm than their non-unionized counterparts, in part, because their ability to strike and bargain collectively allows them to hold up the firm (see Bova 2013). In such a setting, the model suggests that owners are better off offering employees wages only, living with higher inefficient wages, but also retaining some profits, than offering employees an equity stake in the company, negotiating very efficient wages, but retaining none of the profits.

I next assess the impact of equity compensation on production and employee compensation.

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18 For example, Bova et al. (2013) find a negative correlation between union density and the size of equity stakes held by employees and McCarthy et al. (2009) find a monotonically decreasing relationship between unionization and employee ownership.
COROLLARY 1. When the employee ownership equilibrium is supported:

1.) firm production is strictly higher than when both firms’ owners offer wages only.

2.) employee compensation for each group of employees is higher provided 
   \[ \beta > \text{Max}\left[0, \beta^{**}(k)\right], \text{otherwise each group of employees is better off with wages only.} \]

3.) both sets of owners and both sets of employees are worse off than if both sets of owners 
   offered wages only, provided \[ \beta < \text{Min}\left[\text{Max}\left[0, \beta^{**}(k)\right], \beta^*(k)\right]. \]

The inferences from Corollary 1 are consistent with several findings. First, the empirical 
literature finds a correlation between a firm offering equity incentives to its employees and both 
greater firm production (see for example, Hochberg and Lindsay 2010, Kim and Ouimet 2013, 
Jones and Kato 1995). A frequently cited explanation for this correlation is that equity incentives 
drive greater worker effort which, in turn, leads to increased firm production and firm profits. 
However, Corollary 1 provides another explanation for these findings. Greater production may 
also arise because workers agree to more efficient wages. More efficient wages lead to a lower 
cost base for each firm, which in turn lead to greater production.19

Second, the model’s outcomes are consistent with several puzzling findings related to the 
size of employee equity stakes. Specifically, in a publicly-traded setting, when employees have 
smaller equity stakes, they tend to enjoy more of the ensuing surplus (i.e., Kim and Ouimet 
2013). Conversely, all stakeholders appear to be worse off when employees have larger equity 
stakes (Kim and Ouimet 2013, Faley et al. 2006). When \[ \beta > \text{Max}\left[0, \beta^{**}(k)\right], \text{competitive 

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19 This finding builds on the literature that documents a link between compensation and production in a Cournot 
oligopoly. For example, Vickers (1985) and Fershtman and Judd (1987) find that compensating employees based on 
revenues as opposed to profits encourages greater production. Also, Brander and Lewis (1986) find that when a firm 
faces higher fixed costs (e.g., a firm finances with debt instead of equity), managers may have an incentive to 
overproduce. In contrast, the model presented in this paper finds that the firm can generate higher production by 
linking employee compensation to profits when costs are variable (i.e., total labor costs increase with production).
pressures are low and equity stakes are fairly small. In this setting, I find that consistent with the empirical evidence, employees are better off with wages and equity compensation than with wages only. This region is represented by the lightly shaded parameter space to the left of the dashed line in Figure 3. The employees are better off because wages are still relatively large and they enjoy a small share of the firm’s profits when the firm’s profits are also relatively larger due to less competition. Conversely, when $\beta < \max\left[0, \beta^*(k)\right]$, competitive pressures are high and equity stakes are fairly large, and I find that not only are the firm’s owners worse off with the employee ownership equilibrium, but so are the firm’s employees. This region is represented by the dark shaded parameter space to the right of the dashed line in Figure 3. The employees are worse off because their wages are relatively low and although they enjoy a larger share of the firm’s profits, the firm’s profits are also comparatively low because competitive pressures are high.

**Figure 3**
Parameter space over which wages are strictly positive and both firms’ owners offer equity compensation
The collective findings provide a possible explanation for the mixed evidence with respect to the effect of employee ownership on various stakeholders, including a possible insight into how large equity stakes arise endogenously despite both the firm’s owners and employees potentially being worse off when employees are compensated with equity (i.e., the parameter space encompassed by $\beta < \text{Min}\left[\text{Max}\left[0, \beta''(k)\right], \beta'(k)\right]$). Finally, this result also supports the Kim and Ouimet (2013) finding of a negative relationship between employee surplus and the size of employee equity stakes in the company.

3. Limitations and Implications

A possible limitation of the analysis is that the model does not incorporate uncertainty with respect to any of the model’s parameters. On the one hand, not having any uncertainty in the model highlights the potential robustness of the result, as equity compensation continues to arise endogenously despite the lack of uncertainty.\(^{20}\) This result runs in contrast to many models where, in a first-best setting with no uncertainty, a principal would not compensate employees with equity in the company. For example, in the classic Hölmstrom (1979) result, in a first-best world where the agent’s actions are observable, the optimal strategy is to pay the agent a fixed wage provided the agent supplies the desired effort.

On the other hand, in a model without uncertainty, the risk preferences of the employees play no role in the equilibrium. Specifically, absent uncertainty, the value of one dollar of wages is equivalent to one dollar of firm profit, and employees will negotiate the same contracts irrespective of whether they are risk neutral, risk averse, or risk seeking. In practice, however,

\(^{20}\) This modelling approach follows in the spirit of Ray (2007) where the analysis is presented in a “first-best world to show the main forces at work and highlight the key economic intuition.”
because there is variation around the expected payoff of an equity stake, a risk averse employee should value $1 of expected equity returns less than $1 of guaranteed wages. To address this concern, I assess whether the model’s results still hold when employees impose a discount on company profits (i.e., the returns on their equity stakes) relative to their wages by replacing the employees’ objective function in (11) with (16) below:

$$E_i(z_i, w_i, w_j, k, \delta) = q_i(w_i, w_j, k)[w_i - c] + \delta z_i \Pi_i(w_i, w_j, k) \quad i, j = 1, 2, i \neq j$$ (16)

$\delta$ represents the discount that a risk averse employee would place on an expected dollar of equity returns relative to a guaranteed dollar of wages and $\delta \in (0,1)$. I rerun the optimization program in Section 2.2, alternately using values of $\delta = 3/4$, $\delta = 1/2$, and $\delta = 1/4$, and I find that the general tenor of the results still holds. Specifically, below some threshold, $\hat{\beta}(k)$, both firms’ owners offer an equity stake to their employees. The results imply that owners will still offer company stock endogenously, even when employees place a discount on equity-based compensation, provided employee bargaining power is sufficiently low.

Another limitation of the result is that a cooperative solution to the Prisoner’s Dilemma may be achieved if the game is played in an infinitely repeated setting. In other words, both firms’ owners may commit to the cooperative equilibria of offering wages only if negotiations occur infinitely many times. However, a repeated game may not be appropriate for this setting, as the decision to offer an equity stake in one period has the potential to limit compensation choices in future periods. Specifically, offering an equity stake in one period means that, unless the employees decide to divest themselves of company stock, employees will have equity ownership in subsequent periods by construction. This outcome may potentially make moot the
owners’ decision on whether to offer employees wages only or wages and an equity stake in a subsequent period, and in turn prevent the game from being played in a repeated manner.

Finally, a solution to reducing a firm’s cost base when an inefficient supplier sets prices above the competitive market price is for the downstream firm to vertically integrate its rent-extracting supplier. In this way, the downstream firm can acquire its inputs at marginal cost, reduce double marginalization, and boost profits across the channel. While employees cannot be vertically integrated into the firm, one way to potentially make wages more efficient might be to replace a price-setting employee base (i.e., an inefficient supplier of labor) with an employee base that can be paid the competitive market wage. This solution would lower the firm’s cost base and possibly preclude the need to offer employees an equity stake in the company. However, this solution may be impractical for several reasons. First, Lindebeck and Snower (1986, 2001) suggest that inefficiencies in the labor market often arise when employee switching costs are high. By construction, when switching costs are high, replacing existing employees may be unfeasible. Second, due to their ability to hold up the firm, inefficient wages also arise in highly unionized settings. However, in many jurisdictions it is illegal to threaten to replace unionized employees with non-unionized counterparts (who typically accept more efficient wages than their unionized peers). Taken together, replacing a price-setting employee base with a more wage-efficient one may not be a practical alternative to offering existing employees equity-based compensation.

The model also provides some insights on when we might expect firms’ owners to offer their employees company stock. For example, we might expect to see the employee ownership equilibrium arise following negative shocks to employee bargaining power (i.e., negative shocks to $\beta$ that lead to $\beta < \beta^*(k)$). An example of a negative shock to employee bargaining power
might be the Financial Crisis reducing the UAW’s negotiation leverage with the Big 3 automakers during their 2009 contract negotiations. Given the model’s predictions, it is possible that this negative shock was a factor that led to UAW employees being compensated with large equity stakes in their firms.

Additionally, we might expect to see the employee ownership equilibrium arise following positive shocks to industry competitiveness (i.e., positive shocks to \( k \) that lead to \( \beta < \beta^*(k) \)), such as industry deregulation. For example, the adoption of the Airline Deregulation Act in 1978 led to more heated price competition amongst airlines over the following two decades (Cappelli 1985). Interestingly, and consistent with the model’s predictions, by the mid-90s at least 11 major airline carriers compensated employees with significant equity stakes in their respective companies, with United Airlines becoming the first majority employee-owned, publicly traded American airline in 1994.21

4. Conclusion

While equity compensation is frequently lauded as a means to align incentives between owners and employees, it is not clear whether the incentive effect remains when the firm employs a large number of workers. Given this point, it is interesting to note that numerous publicly-traded firms, which employ many workers, offer equity-based compensation to their employees. The literature has posited several explanations, such as retention and sorting, for this outcome. The preceding analysis provides another factor. Specifically, in settings where employee bargaining power is sufficiently low and inter-firm competition is sufficiently intense, we may expect employee ownership to arise not only in specific firms, but also across entire industries.

The analysis also provides a possible explanation for the varied outcomes that arise when employees in publicly-traded companies are compensated with company stock. In particular, comparative statics related to the size of equity stakes provide insights into why, in some cases, employees extract more of the surplus, and in other cases, no stakeholder is better off when an equity stake is offered to employees. The model also provides a possible explanation for several other empirical regularities, such as the positive correlation between employee equity stakes and firm production and the substitutionary relationship between employee ownership and unionization. Finally, the model provides a set of testable predictions regarding the timing of firms’ adoption of equity-based compensation plans. For example, we might expect negative shocks to employee bargaining power or positive shocks to competition intensity to precede the adoption of employee ownership plans for not only specific firms, but also across entire industries.
Appendix

Proof of Proposition 1

For $\forall \beta \in (0,1)$:

$S'(\beta) - S''(\beta) = \frac{\beta^2(1-c)^2}{16} > 0$ (claim 1).

$\Pi'(\beta) - \Pi''(\beta) = -\frac{\beta(4-\beta)(1-c)^2}{16} < 0$ (claim 2).

$E'(\beta) - E''(\beta) = -\frac{\beta^2(1-c)^2}{8} < 0$ (claim 3).

This proves Proposition 1.

Proof of Lemma 1

For $\forall k \in (0,1]$ and $\forall \beta \in (0,1)$:

$\frac{\partial z_i^z(k,\beta)}{\partial k} = \frac{\partial z_i^z(k,\beta)}{\partial k} = (1-\beta)k > 0$.

$\frac{\partial w_i^z(k,\beta)}{\partial k} = \frac{\partial w_i^z(k,\beta)}{\partial k} = -\frac{2(1-c)k(4+k)}{(4+2k-k^2)^2} < 0$.

This proves Lemma 1.

Proof of Proposition 2

For $\forall k \in (0,1]$ and $\forall \beta \in (0,1)$:

$\Pi_i^z(\beta,k) - \Pi_i^{zw}(\beta,k) = 4(1-c)^2 \left( \frac{4(-2\beta - (1-\beta)k^2)(-8 + (4+k)k + 2\beta(1+k))}{(2+k)(4-\beta k)^2(4+2k-k^2)^2} \right) > 0$ (claim 3).

$S_i^z(\beta,k) - S_i^{zw}(\beta,k) = \frac{2(1-c)^2(2\beta + (1-\beta)k^2)A}{(2+k)(4-\beta k)^2(32-16k^2 + \beta k^4)^2}$

Where:

$A = -(1024\beta - 512(1+\beta^3)k^2 + 16(16+\beta(-16+\beta(20+(-1+\beta)\beta))))k^4 - 8\beta^2(3+\beta^2)k^6 + \beta^4k^8$
$S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k)$ is concave in $\beta$ for $\forall \beta \in (0, 1)$ and $\forall k \in (0, 1]$. Solving for $\beta$ so that $S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k) = 0$ produces several roots. However, only one root, $\beta^*(k)$, is positive for $\forall k \in (0, 1]$. Moreover, $\beta^*(k) \in (0, 1)$ for $\forall k \in (0, 1]$. I define $\beta^*(k)$ as follows:

$$C = (452984832k^4 - 570425344k^6 + 50331648k^8 + 26345720k^{10} - 180420608k^{12} + 52789248k^{14} - 7446528k^{16} + 414720k^{18} + (205195258022068224k^8 - 516788057240764416k^{10} + 25031921522595264k^{12} + 676621863547305984k^{14} - 1387027321369657344k^{16} + 132082572621264384k^{18} - 799885225919250432k^{20} + 336297794702671872k^{22} - 101528091960016896k^{24} + 22208751901605888k^{26} - 3489106608783360k^{28} + 382859156127744k^{30} - 27729919475712k^{32} + 1182290411520k^{34} - 22196256768k^{36})^{1/2}/13$$

$$D = \frac{C}{3(2^{1/3}k^4(4-k^2)^2)} - \frac{8(64k^2 - 40k^4 + 3k^6)}{3(16k^4 - 8k^6 + k^8)} + \frac{(64(2^{1/3}(4864k^4 - 6656k^6 + 3328k^8 - 720k^{10} + 57k^{12}))}{(3k^4(4-k^2)^2} C)$$

$$F = \frac{8(64 - 40k^2 + 3k^4)}{k^2(4-k^2)^2}$$

$$\beta^*(k) = \frac{1}{2} \left( \sqrt{\frac{8}{(4-k^2)^2}} - \frac{D + F + \frac{64}{(4-k^2)^4}}{\sqrt{-D + F + \frac{128}{(4-k^2)^4} + \frac{128(2-k^2)(2048 - 1536k^2 + 60k^4 + 198k^8 - 53k^8 + 4k^{10})}{k^4(4-k^2)^6}} \right)$$

Finally, $S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k)\bigg|_{\beta=0} = \frac{(1-c)^2k^4}{8(2+k)^2(2-k^2)} > 0$ for $\forall k \in (0, 1]$. Given this point, and $S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k)$ is concave in $\beta$ for $\forall \beta \in (0, 1)$, and $\beta^*(k) \in (0, 1)$ for $\forall k \in (0, 1]$, then $S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k) > 0$ provided $\beta < \beta^*(k)$. Thus, when $\beta < \beta^*(k)$, the best response for firm 1’s owners when firm 2’s owners offer wages only is to offer employees wages and an equity stake in the firm.

From the previous point, we know that, $S_{1w}^z(\beta, k) - S_{1w}^w(\beta, k) > 0$ provided $\beta < \beta^*(k)$. Thus, if $(S_{2z}^z(\beta, k) - S_{2w}^z(\beta, k)) - (S_{1z}^z(\beta, k) - S_{1w}^w(\beta, k)) > 0$, firm 2’s owners’ best response to firm 1’s owners offering both wages and an equity stake is to also offer both wages and an equity stake provided $\beta < \beta^*(k)$, as this would imply that $(S_{2z}^z(\beta, k) - S_{2w}^z(\beta, k)) > 0$. 

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Let: 
\[ (S_2^\beta(\beta,k) - S_2^{\beta w}(\beta,k)) - (S_1^{\beta w}(\beta,k) - S_1^{\beta w}(\beta,k)) = \frac{G(H + I + J)}{K} \]

Where, for \( \forall k \in (0,1] \) and \( \forall \beta \in (0,1) \):

\[ G = 16(1-c)^2 k(2\beta + (1-\beta)k^2) > 0 \]
\[ H = \beta^3 k^3 (2+k)^3 (4-2k^2 + k^3) + 16k(32 + k(16 + k(-24 - (12-2)k))) > 0 \]
\[ I = -2\beta^2 k(32 + k(32 + k^2(2+k)(-4 + 3k(2+k))) < 0 \]
\[ J = 8\beta(64 + k^2 (2+k)(-40 + k(20 + k(8+k))) > 0 \]
\[ K = (2+k)^3 (4-\beta k)^2 (4+2k-k^2)^2 (32 - 16k^2 + \beta k^4)^2 > 0 \]

For \( \forall k \in (0,1] \), \( I + J \) is concave in \( \beta \) with two roots. The first root equals zero. The second root equals \( \frac{4(64-80k^2 + 36k^4 + 10k^5 + k^6)}{k(32 + 2k - 8k^3 + 8k^4 + 12k^5 + 3k^6)} > 1 \) \( \forall k \in (0,1] \).

Thus, \( I + J > 0 \) \( \forall \beta \in (0,1) \) and \( \forall k \in (0,1] \).

It follows that, \( (S_2^\beta(\beta,k) - S_2^{\beta w}(\beta,k)) - (S_1^{\beta w}(\beta,k) - S_1^{\beta w}(\beta,k)) > 0 \) for \( \forall k \in (0,1) \) and \( \forall \beta \in (0,1) \). Thus, when it is more profitable for firm 1’s owners to offer wages and an equity stake as a best response to firm 2’s owners offering wages only (i.e., \( \beta < \beta^*(k) \)), firm 2’s owners will also be better off in offering a wage and an equity stake as a best response to firm 1’s owners offering a wage and an equity stake. This outcome results in a unique, pure strategy equilibrium where both firms offer wages and equity stakes to their employees provided \( \beta < \beta^*(k) \) (claim 1).

Let: 
\[ S_1^{\beta w}(\beta,k) - S_1^{\beta w}(\beta,k) = \frac{2(l-c)^2 (2\beta + (1-\beta)k^2) L}{(4-\beta k)^2 (8 + 8k - k^3)^2} \]

Where for \( \forall k \in (0,1] \) and \( \forall \beta \in (0,1) \):

\[ L = \beta^2 k^2 (2+k)^2 + 8k(4+3k) - 8\beta(-2 + k(2+k(4+k))) > 0 \]

Thus, \( S_1^{\beta w}(\beta,k) - S_1^{\beta w}(\beta,k) > 0 \) for \( \forall k \in (0,1] \) and \( \forall \beta \in (0,1) \) (claim 2).

This proves Proposition 2.

**Proof of Corollary 1**

For \( \forall k \in (0,1] \) and \( \forall \beta \in (0,1) \):
\[
q_i^w(\beta, k) - q_i^{ww}(\beta, k) = q_2^w(\beta, k) - q_2^{ww}(\beta, k) = \frac{4(1-c)(2\beta + (1-\beta)k^2)}{(4-\beta k)(8+8k-k^3)} > 0 \quad \text{(claim 1)}.
\]

For \( \forall k \in (0,1] \) and \( \forall \beta \in (0,1) \), \( E_i^\bar{\bar{w}}(\beta, k) - E_i^{ww}(\beta, k) \) is convex in \( \beta \) with two roots. The first root is negative for \( \forall k \in (0,1] \). The second root, \( \beta^*(k) = \frac{-2(8 - 4k - 6k^2 + k^3)}{k^2(2+k)} \). \( \beta^*(k) \) is increasing in \( k \) and can be negative, zero, or positive over \( k \in (0,1] \). When \( \beta^*(k) \leq 0 \), \( E_i^\bar{\bar{w}}(\beta, k) - E_i^{ww}(\beta, k) > 0 \). When \( \beta^*(k) > 0 \), \( E_i^\bar{\bar{w}}(\beta, k) - E_i^{ww}(\beta, k) > 0 \) provided \( \beta > \beta^*(k) \). Thus, \( E_i^\bar{\bar{w}}(\beta, k) - E_i^{ww}(\beta, k) > 0 \) when \( \beta > \max\left[0, \beta^*(k)\right] \) (claim 2). Claim 3 follows from claims 1 and 2 in Proposition 2 and claim 2 of Corollary 1. This proves Corollary 1.
REFERENCES


